بسم الله الرحمن الرحيم الدورة العادية

الحل النموذجي لامتحان الفيزياء الإحصائية

حل التمرين الأول:-

1) حساب دالة التجزئة $Z_1(T,L)$

$$Z_1 = \frac{1}{h}\int_{-\infty}^{+\infty} dp\, e^{-\beta\frac{p^2}{2m}} \int_0^L dq\; e^{+\beta\alpha \ln\left(\frac{q}{L_0}\right)}$$

$$= \frac{1}{h}\left(\frac{2\pi m}{\beta}\right)^{1/2} \int_0^L e^{\ln\left(\frac{q}{L_0}\right)^{\alpha\beta}} dq$$

$$= \frac{1}{h}\left(\frac{2\pi m}{\beta}\right)^{1/2} \int_0^L \frac{1}{L_0^{\alpha\beta}}\, q^{\alpha\beta}\, dq$$

$$= \frac{1}{h}\left(\frac{2\pi m}{\beta}\right)^{1/2} \frac{1}{L_0^{\alpha\beta}}\left(\frac{q^{\alpha\beta+1}}{\alpha\beta+1}\right)\Bigg|_0^L$$

$$\Rightarrow \quad Z_1(T,L) = \frac{1}{h}\left(\frac{2\pi m}{\beta}\right)^{1/2} \frac{1}{L_0^{\alpha\beta}}\left(\frac{L^{\alpha\beta+1}}{\alpha\beta+1}\right) \qquad (5\,pts)$$

2) عبارة الطاقة الحرة F للنظام

$$F = -kT \ln\left[Z_1\right]^N$$

$(1,5\,pt)$

$$F = -NkT \ln Z_1$$

$$= -NkT\left[\ln\left(\frac{1}{h}\left(\frac{2\pi m}{\beta}\right)^{1/2}\frac{1}{L_0^{\alpha\beta}}\right) + \ln\left(\frac{L^{\alpha\beta+1}}{\alpha\beta+1}\right)\right]$$

$$= -NkT\left[\ln(\ldots) + (\alpha\beta+1)\ln L - \ln(\alpha\beta+1)\right]$$

3) حساب عبارة الضغط

$$P = -\left(\frac{\partial F}{\partial L}\right) = +NkT\frac{\partial}{\partial L}(\alpha\beta+1)\ln L$$

$$\Rightarrow \quad P = \frac{NkT}{L}(\alpha\beta+1) \qquad (3\,pts)$$

(B4 حساب الضغط لما $T \to 0$

$$P = \frac{NkT}{L}\left(\frac{\alpha}{kT} + 1\right) \qquad \beta = \frac{1}{kT}$$

(1,5 pts) $$= \frac{N}{L}\left(\alpha + kT\right) \Longrightarrow T \to 0 \quad P = \frac{N\alpha}{L}$$

$\alpha = 0$ ⟸ $P = 0$ لما نضع α يساوي الصفر فإن مساهمة الضغط الناتجة عن الكمون سوف تختفي (1pt)

5) حساب الطاقة الداخلية للنظام $\bar{E}$:

$$\bar{E} = -\frac{\partial \ln Z}{\partial \beta} = -N\frac{\partial}{\partial\beta}\ln Z_1$$

$$\bar{E} = -N\frac{\partial}{\partial\beta}\Big[\ln\left(\frac{1}{h}(2\pi m)^{\frac{1}{2}}\right) - \frac{1}{2}\ln\beta - \alpha\beta\ln L_0 + (\alpha\beta+1)\ln L - \ln(\alpha\beta+1)\Big]$$

$$= -N\frac{\partial}{\partial\beta}\Big[\ln(\dots) - \frac{1}{2}\ln\beta + \alpha\beta\ln\left(\frac{L}{L_0}\right) + \ln L - \ln(\alpha\beta+1)\Big]$$

$$= -N\left[-\frac{1}{2\beta} + \alpha\ln\frac{L}{L_0} - \frac{\alpha}{\alpha\beta+1}\right]$$

$$\bar{E} = \frac{N}{2\beta} + \frac{N\alpha}{\alpha\beta+1} - N\alpha\ln\left(\frac{L}{L_0}\right)$$ (3 pts)

حل التمرين 2 :

$$\hat{\rho} = |\psi\rangle\langle\psi| = \left(\frac{\sqrt{3}}{2}|+\rangle + \frac{1}{2}|-\rangle\right)\left(\langle+|\frac{\sqrt{3}}{2} + \langle-|\frac{1}{2}\right)$$

$$= \frac{3}{4}|+\rangle\langle+| + \frac{\sqrt{3}}{4}|+\rangle\langle-| + \frac{\sqrt{3}}{4}|-\rangle\langle+| + \frac{1}{4}|-\rangle\langle-|$$

نكتبها على الشكل المصفوفي حيث $|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ و $|-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ فنجد

$$\hat{\rho} = \begin{pmatrix}\frac{3}{4} & \frac{\sqrt{3}}{4}\\ \frac{\sqrt{3}}{4} & \frac{1}{4}\end{pmatrix}$$ (2 pts)

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حساب القيم المتوسطة لـ $\hat{S}_x$ و $\hat{S}_z$

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle \hat{S}_x \rangle = Tr\left\{\hat{\rho}\hat{S}_x\right\} = Tr\begin{pmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{\hbar}{2} & 0 \end{pmatrix}$$

$$= Tr\begin{pmatrix} \frac{\sqrt{3}}{8}\hbar & \frac{3}{8}\hbar \\ \frac{\hbar}{8} & \frac{\sqrt{3}}{8}\hbar \end{pmatrix} = \frac{\sqrt{3}}{4}\hbar$$

(1,5 pt)

$$\langle \hat{S}_z \rangle = Tr\left\{\hat{\rho}\hat{S}_z\right\} = Tr\begin{pmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{pmatrix}$$

$$= Tr\begin{pmatrix} \frac{3\hbar}{8} & -\frac{\sqrt{3}}{8}\hbar \\ \frac{\sqrt{3}}{8}\hbar & -\frac{\hbar}{8} \end{pmatrix} = \frac{\hbar}{4}$$

(1,5 pt)