التحويلات المالتية
3M فدوى ...

التمرين الأول :

- $L(2t^2-1)(p) = 2L(t^2)_{(p)} - L(1)(p) = 2 \cdot \frac{2!}{p^{2+1}} - \frac{1}{p}$

(ن 02) $= \frac{4}{p^3} - \frac{1}{p} = \frac{4-p^2}{p^3}$; $Re(p) > 0 = p_0$

لدينا :
- $L(3e^t - e^{3t}\cdot\cos(\frac{2}{3}t))(p) = 3\cdot L(e^t)_{(p)} - L(e^{3t}\cos(\frac{3}{2}t))(p)$

$= 3L(e^t)(p) - L(\cos\frac{3}{2}t)(p-3)$

(ن 03) $= 3\cdot\frac{1}{p-1} - \frac{p-3}{(p-3)^2+\frac{4}{9}}$; حيث $\left(L(\cos\frac{3}{2}t)(p) = \frac{3}{2}L(\cos t)(\frac{3}{2}p) = \frac{3}{2}\cdot\frac{(\frac{3}{2}p)}{(\frac{3}{2}p)^2+1} = \frac{p}{p^2+\frac{4}{9}}\right)$

$= \frac{3}{p-1} - \frac{9(p-3)}{9(p-3)^2+4}$.

$Re(p) > 3 = p_0$

(ن 1) $L(h*g)(p) = L(h)(p)\cdot L(g)(p) = \left(\frac{4-p^2}{p^3}\right)\left(\frac{3}{p-1} - \frac{9(p-3)}{9(p-3)^2+4}\right)$

$Re(p) > 3 = p_0$

التمرين الثاني :

① $L(p) = \frac{a}{p-2} + \frac{bp+c}{p^2+1}$; $a = \lim_{p\to 2}(p-2)L(p) =$

$\lim_{p\to 2}\frac{2p+1}{p^2+1} = \frac{5}{5} = 1$

(ن 03) $\lim_{p\to i}(p-i)L(p) = \lim_{p\to i}\frac{bp+c}{p+i} \Leftrightarrow$

$= \frac{bi+c}{2i} \Leftrightarrow c+bi = \frac{2i+1}{i-2}$

$\frac{2i+1}{(i-2)(2i)}$

$= \frac{(1+2i)(-2-i)}{(-2+i)(-2-i)} = \frac{-5i}{5} = -i$

إذا $b=-1$, $c=0$

$L(p) = \frac{1}{p-2} - \frac{p}{p^2+1}$

② $L(f_1(t))(p) = L(e^{-2t}f(t))$

$= L(f(t))(p+2)$ (1,0 ن)

$= \frac{2(p+2)+1}{(p+2-2)((p+2)^2+1)} = \frac{2p+5}{p(p^2+4p+5)}$

$L(f_2(t))(p) = L(t\cdot f(t))(p)$

$= -(L(p))' = +\frac{1}{(p-2)^2}$ (1,0 ن)

$+\frac{p^2+1-2p^2}{(p^2+1)^2} = \frac{1}{(p-2)^2} + \frac{p^2-1}{(p^2+1)^2}$

(1,2) $L(f_3(t))(p) = 2\cdot L\left(f(\frac{1}{2}t)\right) = 2\,L(f(t))(2p) = 2\cdot\frac{4p+1}{(2p-1)(4p^2+1)}$

(1,3) $L(f_4(t))(p) = 2\cdot L(f(t-1))(p) + 3\cdot L(f(t+1))(p)$

$$= \left(2e^{-p}+3e^{p}\right)L(f(t))(p)$$

$$= \frac{(2e^{-p}+3e^{p})(2p+1)}{(p-2)(p^2+1)}$$

(3

$$f(t) = e^{2t} - \cos t$$

02

(4) بإدخال تحويل لابلاس على طرفي المعادلة نجد

$$\left[p^2 L(y)_{(p)} - \underbrace{y'(0)}_{=2} - p\cdot \underbrace{y(0)}_{=0}\right] - \frac{5}{2}\left[p\,L(y)(p) - y(0)\right] + L(y)(p)$$

$$= -\frac{5}{2}\cdot\frac{1}{p^2+1}$$

03

ومنه

$$\left(p^2-\frac{5}{2}p+1\right)L(y)(p) = -\frac{5}{2}\frac{1}{p^2+1}+2 = \frac{1}{2}\left(\frac{-5+4p^2+4}{p^2+1}\right)$$

$$L(y)(p) = \frac{4p^2-1}{\underbrace{(2p^2-5p+2)}_{(2p-1)(p-2)}(p^2+1)} = \frac{2p+1}{(p-2)(p^2+1)}$$

$$= L\left(e^{2t}-\cos t\right)(p)$$

إذا :

$$y(t) = e^{2t} - \cos t$$