Exo1 :

$$\vec{\nabla}(fg) = \frac{\partial}{\partial x}(fg)\vec{i} + \frac{\partial}{\partial y}(fg)\vec{j} + \frac{\partial}{\partial z}(fg)\vec{k}$$

$$= f\frac{\partial g}{\partial x}\vec{i} + g\frac{\partial f}{\partial x}\vec{i} + f\frac{\partial g}{\partial y}\vec{j} + g\frac{\partial f}{\partial y}\vec{j} + f\frac{\partial g}{\partial z}\vec{k} + g\frac{\partial f}{\partial z}\vec{k}$$

$$= f\left(\frac{\partial g}{\partial x}\vec{i} + \frac{\partial g}{\partial y}\vec{j} + \frac{\partial g}{\partial z}\vec{k}\right) + g\left(\frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}\right)$$

$$= f\vec{\nabla}g + g\vec{\nabla}f$$

Exo2

$\vec{E} = z\rho\cos^2\varphi\,\vec{u_z}$. On a, $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ avec $\varepsilon_0 = 1$

$$\Rightarrow \rho = \vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial z}(z\rho\cos^2\varphi) = \boxed{\rho\cos^2\varphi}$$

au point $(1, \pi/4, 3)$, la densité de charge électrique est :

$$\rho = (1)\cos^2\left(\frac{\pi}{4}\right) = 0{,}5\ C/m^3$$

- La charge totale, $Q = \int_V \rho\,dV = \int_V \rho\cos^2(\varphi)\,\rho\,d\rho\,d\varphi\,dz$

$$= \int_{-2}^{2} dz \int_0^{2\pi} \cos^2(\varphi)\,d\varphi \int_0^1 \rho^2\,d\rho = \boxed{\frac{4\pi}{3}\,C}$$