

تصحيح امتحان الدورة العادية لمقياس رياضيات 1

. التمرين 1: (10 نقاط)

$$A = \frac{1}{2} + \frac{n}{n-1} = \frac{3}{2} + \frac{1}{n-1} \quad \text{نقاط 4}$$

$$n > 1 \Leftrightarrow n - 1 > 0 \Leftrightarrow 0 < \frac{1}{n-1} \leq 1 \Leftrightarrow \frac{3}{2} < \frac{3}{2} + \frac{1}{n-1} \leq \frac{5}{2}$$

$$\sup A = \frac{5}{2}, \max A = \frac{5}{2}, \inf A = \frac{3}{2}, \min A \text{ غير موجود}$$

$$(2) \text{ نضع } y = \frac{1}{x} \text{ ومنه } x \rightarrow +\infty \Rightarrow y \rightarrow 0 \quad \text{نقاط 3}$$

$$\lim_{x \rightarrow +\infty} x \left[\ln \left(1 + \frac{1}{x} \right) \right] = \lim_{y \rightarrow 0} \frac{\ln(1+y)}{y} = 1$$

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x} \right)^x = \lim_{x \rightarrow 0} e^{\ln(1+\frac{1}{x})^x} = \lim_{x \rightarrow 0} e^{x \ln(1+\frac{1}{x})} = e^1 = e \quad \text{نقطة 3}$$

$$\text{التمرين 2: (6 نقاط) نضع } f(x) = \text{Arcsin} \left(\frac{x+1}{x-2} \right) \quad \text{نقاط 3}$$

$$\frac{x+1}{x-2} = 1 - \frac{1}{x-2}$$

$$-1 \leq x \leq 1 \Rightarrow -3 \leq x-2 \leq -1 \Rightarrow -1 \leq \frac{1}{x-2} \leq -\frac{1}{3}$$

$$\Rightarrow 0 \leq 1 - \frac{1}{x-2} \leq \frac{2}{3}$$

$$D_f = \left[0, \frac{2}{3} \right]$$

$$(2) \text{ احسب القيم التالية } \text{Arctan}(\sqrt{3}), \text{Arcsin}(1), \text{Arccos}(0) \quad \text{نقاط 3}$$

$$\text{Arccos}(0) = x \Leftrightarrow \cos x = 0 \Leftrightarrow x = \frac{\pi}{2}$$

$$\text{Arcsin}(1) = x \Leftrightarrow \sin x = 1 \Leftrightarrow x = \frac{\pi}{2}$$

$$\text{Arctan}\sqrt{3} = x \Leftrightarrow \tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3}$$

التمرين 3 (6 نقاط)

$$\begin{aligned}
 \text{3 نقاط} \dots \dots \dots \ker f &= \{(x, y) \in \mathbb{R}^2 : f(x, y) = 0_{\mathbb{R}^3}\} \\
 &= \{x \in \mathbb{R}^2 : (x - 2y, x + y, x - y) = (0, 0, 0)\} \\
 &= \{x \in \mathbb{R}^2 : x - 2y = 0, x + y = 0, x - y = 0\} \\
 &= \{x \in \mathbb{R}^2 : x = 2y, x = y, x = y\} \\
 &= \{x \in \mathbb{R}^2 : x = y = 0\} \\
 &= \{(0, 0, 0)\} \\
 &= \langle (0, 0, 0) \rangle
 \end{aligned}$$

$$\begin{aligned}
 \text{3 نقاط} \dots \dots \dots \text{Im} f &= \{f(x, y) : (x, y) \in \mathbb{R}^2\} \\
 &= \{(x - 2y, x + y, x - y) : (x, y) \in \mathbb{R}^2\} \\
 &= \{(x, x, x) + (y, -2y, y) : (x, y) \in \mathbb{R}^2\} \\
 &= \{x(1, 1, 1) + y(1, -2, 1) : (x, y) \in \mathbb{R}^2\} \\
 &= \langle (1, 1, 1), (1, -2, 1) \rangle
 \end{aligned}$$