# Correction de controle 2019/2020
# Programmation linéaire

Exo 1 :

1) Oui le pb admet un plan de transport optimal car la condition de balance est vérifié $\sum a_i = \sum b_j = 700 + 400 + 500 = 800 + 800 = 1600$ (1)

2) CNW

| | $V_1$ | $V_2$ | $a_i$ |
|---|---|---|---|
| $C_1$ | 20 / 700 | 25 | 700 |
| $C_2$ | 15 / 100 | 10 / 300 | 400 |
| $C_3$ | 10 | 15 / 500 | 500 |
| $b_j$ | 800 | 800 | 1600 |

(1)

$\text{Coûts} = 700 \times 20 + 100 \times 15 + 300 \times 10 + 500 \times 15 = 26000$

EM

| | $V_1$ | $V_2$ | $a_i$ |
|---|---|---|---|
| $C_1$ | 20 / 300 | 25 / 400 | 700 |
| $C_2$ | 15 | 10 / 400 | 400 |
| $C_3$ | 10 / 500 | 15 | 500 |
| $b_j$ | 800 | 800 | 1600 |

(1)

$\text{coûts} = 300 \times 20 + 400 \times 25 + 400 \times 10 + 500 \times 10 = 25000$

3) Plan optimal

| | $V_1$ | $V_2$ | $a_i$ | $u_i$ |
|---|---|---|---|---|
| $C_1$ | 20 / 700 | 25 / -10 | 700 | 0 |
| $C_2$ | 15 / − 100 | 10 / + 300 | 400 | -5 |
| $C_3$ | 10 / + 10 | 15 / − 500 | 500 | 0 |
| $b_j$ | 800 | 800 | 1600 | |
| $v_j$ | 20 | 15 | | |

$\theta_1 = \min(100, 500) = 100$

(2)

| | $V_1$ | $V_2$ | $a_i$ | $u_i$ |
|---|---|---|---|---|
| $C_1$ | 20 / 700 | 25 / 0 | 700 | 0 |
| $C_2$ | 15 / -10 | 10 / 400 | 400 | -15 |
| $C_3$ | 10 / 100 | 15 / 400 | 500 | -10 |
| $b_j$ | 800 | 800 | 1600 | |
| $v_j$ | 20 | 25 | | |

(1,5)

tous $\Delta_{ij} \leq 0$ Donc

coûts min = 25000

$x_{11} = 700$ $\quad x_{21} = 0$ $\quad x_{31} = 100$

$x_{12} = 0$ $\quad x_{22} = 400$ $\quad x_{32} = 400$

(0,5)

| | $x_1$ | $x_2$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $a_1$ | $a_2$ | $b$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | -2 | 0 | 5 | 2 | -5 | 10 |
| $x_2$ | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 1 | 2 |
| $e_3$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | 4 |
| $x_1$ | 1 | 0 | 0 | 1/2 | 0 | -1/2 | -1/2 | 1/2 | 1 |
| $\Delta_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

(1)

tous $\Delta_j \geq 0$ et $W = 0$ donc $Z$ admet un SBR (0,25)

VB : $e_1, x_2, e_3, x_1$
VHB : $e_2, e_4$ (0,25)

$x_2 - e_4 = 2 \Rightarrow x_2 = e_4 + 2$

$x_1 + \frac{1}{2}e_2 - \frac{1}{2}e_4 = 1 \Rightarrow x_1 = -\frac{1}{2}e_2 + \frac{1}{2}e_4 + 1$

$Z = 40x_1 + 25x_2 = 40\left(-\frac{1}{2}e_2 + \frac{1}{2}e_4 + 1\right) + 25(e_4 + 2) = -20e_2 + 45e_4 + 90$ (0,5)

$Z + 20e_2 - 45e_4 = 90$

| | $x_1$ | $x_2$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $b$ |
|---|---|---|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | -2 | 0 | 5 | 10 |
| $x_2$ | 0 | 1 | 0 | 0 | 0 | -1 | 2 |
| $e_3$ | 0 | 0 | 0 | 0 | 1 | 1 | 4 |
| $x_1$ | 1 | 0 | 0 | 1/2 | 0 | -1/2 | 1 |
| $\Delta_j$ | 0 | 0 | 0 | 20 | 0 | -45 | 90 |

(0,5)

| | $x_1$ | $x_2$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $b$ |
|---|---|---|---|---|---|---|---|
| $e_4$ | 0 | 0 | 1/5 | -2/5 | 0 | 1 | 2 |
| $x_2$ | 0 | 1 | 1/5 | -2/5 | 0 | 0 | 4 |
| $e_3$ | 0 | 0 | -1/5 | 2/5 | 1 | 0 | 2 |
| $x_1$ | 1 | 0 | 1/10 | 3/10 | 0 | 0 | 2 |
| $\Delta_j$ | 0 | 0 | 9 | 2 | 0 | 0 | 180 |

(1)

tous $\Delta_j \geq 0$ donc $Z^* = 180$, $x_1 = 2$, $x_2 = 4$ (0,5)

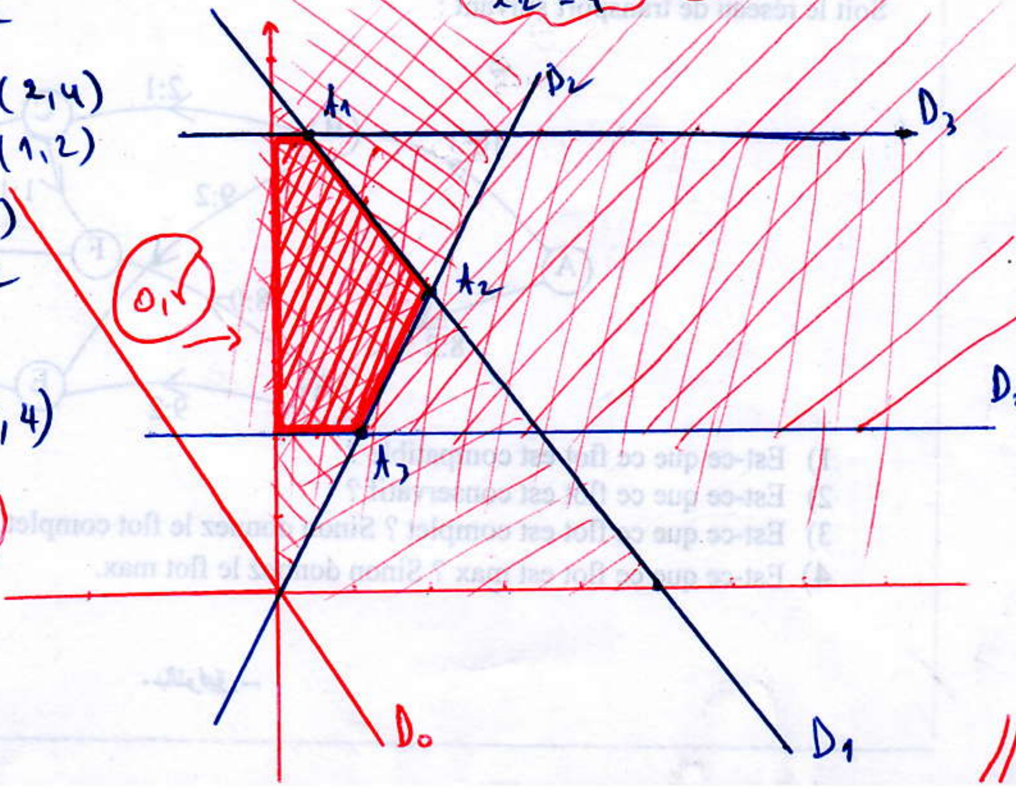
2) Représentation graphique

(0,5) $D_1 : 4x_1 + 3x_2 = 20$ (5,0), (2,4)

(0,5) $D_2 : -2x_1 + x_2 = 0$ (0,0) (1,2)

(0,5) $D_3 : x_2 = 6$ (0,6) (1,6)

(0,5) $D_4 : x_2 = 2$ (0,2) (1,2)




$D^0 = 40x_1 + 25x_2 = 0$ (0,0) (-25, 4)

$A_1(0{,}5, 6) \rightarrow Z = 170$

$A_2(2,4) \Rightarrow Z = 180$

$A_3(1,2) \Rightarrow Z = 90$ (0,5)

Donc $Z^* = 180$, $x_1 = 2$, $x_2 = 4$ (0,5)

Exo2 :

On pose $x_1$ : nbre de bureau
$x_2$ : nbre de fauteuils

$$\text{Max } Z = 40x_1 + 25x_2$$
$$4x_1 + 3x_2 \leq 20$$
$$-2x_1 + x_2 \geq 0$$
$$x_2 \leq 6$$
$$x_2 \geq 2$$
$$x_1 \geq 0,\ x_2 \geq 0$$

Ou

$$\text{Max } Z = 40x_1 + 25x_2$$
$$4x_1 + 3x_2 \leq 20$$
$$2x_1 - x_2 \leq 0$$
$$x_2 \leq 6$$
$$x_2 \geq 2$$
$$x_1 \geq 0,\ x_2 \geq 0$$

2

1) Simplexe a deux phases

$$\text{Max } W = -a_1 - a_2$$
$$4x_1 + 3x_2 + e_1 = 20$$
$$-2x_1 + x_2 - e_2 + a_1 = 0$$
$$x_2 + e_3 = 6$$
$$x_2 - e_4 + a_2 = 2$$
$$x_1, x_2, e_1, e_2, e_3, e_4, a_1, a_2 \geq 0$$

1

$(x_1, x_2, e_1, e_2, e_3, e_4, a_1, a_2) = (0, 0, 20, 0, 6, 0, 0, 2)$ SBR 0,25

VHB : $x_1\ x_2,\ e_2,\ e_4$
VB : $e_1\ a_1,\ e_3,\ a_2$ 0,25

$$-a_1 = -2x_1 + x_2 - e_2$$
$$-a_2 = x_2 - e_4 - 2$$
$$\Rightarrow W = -a_1 - a_2 = 2x_2 - 2x_1 - e_2 - e_4 - 2$$
$$W + 2x_1 - 2x_2 + e_2 + e_4 = -2$$

0,75

| | $x_1$ | $x_2$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $a_1$ | $a_2$ | b |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 4 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 20 |
| $a_1$ | -2 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 |
| $e_3$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 6 |
| $a_2$ | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 1 | 2 |
| $\Delta_j$ | 2 | -2 | 0 | 1 | 0 | 1 | 0 | 0 | -2 |

0,5

| $x_1$ | $x_2$ | $e_1$ | $e_2$ | $e_3$ | $e_3$ | $e_4$ | $a_1$ | $a_2$ | b |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 10 | 0 | 1 | 3 | 0 | 0 | -3 | 0 | 20 |
| $x_2$ | -2 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 |
| $e_3$ | 2 | 0 | 0 | 1 | 1 | 0 | -1 | 0 | 6 |
| $a_2$ | 2 | 0 | 0 | 1 | 0 | -1 | -1 | 1 | 2 |
| $\Delta_j$ | -2 | 0 | 0 | -1 | 0 | 1 | 2 | 0 | -2 |

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