Fluctuation properties and effective temperature of strongly interacting 1d bosons after a quench

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Abstract

We make use of the exact mapping of hardcore bosons (HCB) onto free fermions to investigate their fluctuations properties in momentum space, both in the equilibrium Gibbs ensemble (GE), as well as in the generalized Gibbs ensemble (GGE) describing the long-time evolution after a quantum quench. For the system at equilibrium we test the validity of a fluctuation-dissipation relation connecting the momentum distribution gradient with noise correlations in momentum space. This relation offers a fundamental tool for primary thermometry in weakly interacting, homogeneous systems, and it is found to be useful for the thermometry of hardcore bosons as well, over a broad temperature range. We then turn to the GGE description of the post-quench stationary state, showing that a similar thermometric scheme can provide a close estimate of the effective temperature of the system, which is generally defined by matching the internal energy of the system after the quench with the thermal one. Our results demonstrate the effectiveness of primary noise thermometry in the GGE without previous knowledge of the equation of state of the target Hamiltonian, and offer detailed insights into the fluctuation properties of non-equilibrium stationary states realized by strongly correlated quantum systems.

1. Introduction

The 1D HCB Hamiltonien can be written as:

$$H = -J\sum_i (b_i^+ b_{i+1} + H.c.) + \sum_i V_i n_i \quad (1)$$

Where b_i^+ and b_i are the bosonic operators whith the on-site constraints:

$$b_i^{+2} = b_i^2 = 0, \qquad \{b_i, b_i^+\} = 1, \quad (2)$$

In order to exactly caculate the HCB properties, we use the Jordon-Wigner transformation:

$$b_i^+ = f_i^+ \prod_{\beta=1}^{i-1} exp(-i\pi f_\beta^+ f_\beta), \qquad b_i = \prod_{\beta=1}^{i-1} exp(i\pi f_\beta^+ f_\beta) f_i \quad (3)$$

f_i^+ and f_i are the creation and annihilaiton operators for spinless fermions.

This transformation maps the HCB Hamiltonien onto the one of noninteracting spinless fermions:

$$H = -t\sum_i (f_i^+ f_{i+1} + H.c.) + \sum_i V_i n_i^f \quad (4)$$

The momentum distribution function is:

$$< n_k > = \frac{1}{L}\sum_{ij} e^{-ik(x_i - x_j)} \rho_{ij}, \qquad \rho_{ij} = \langle b_i^+ b_j \rangle \quad (5)$$

The noise correlations are defined as:

$$\Delta_{kk'} = < \delta n_k \delta n_{k'} > = < n_k n_{k'} > - < n_k > < n_{k'} > \quad (6)$$

The relation connecting the momentum distribution gradient whith noise correlations is:

$$-k_B T \frac{\partial \langle n_k \rangle}{\partial k} \simeq J \sum_{k'} 2\Delta_{kk'} \sin k' \quad (7)$$

this relation is found to be useful for the thermometry of HCB over a broad temeprature range. Let us take:

$$A_k = \sum_{k'} 2\Delta_{kk'} \sin k' \qquad and \qquad B_k = -\frac{\partial \langle n_k \rangle}{\partial k} \quad (8)$$

2. Thermometry of HCB in GE

The estemated temperature T_{est} can be found using:

$$k_B T_{est} B_{k(GE)} \simeq J A_{k(GE)} \quad (9)$$

where:

$$\langle O \rangle_{GE} = \frac{1}{Z_{GE}} Tr\left[O e^{-\beta H}\right], \qquad Z_{GE} = Tr\left[e^{-\beta H}\right], \qquad \beta = \frac{1}{k_B T} \quad (10)$$

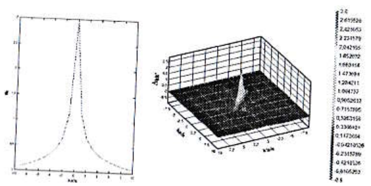

Fig. 1: *Momentum distribution function and Noise correlations in a half-filled system for $T = 0.1J$ and $L = 20$.*

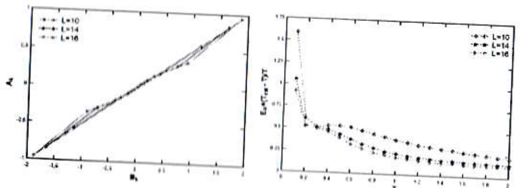

Fig. 2: *Left: A_k as a function of B_k in GE for $T = 0.3J$. Right: Relative error between the estimated temperature - equation ?? - and the broad temperature in GE.*

3. Thermometry of HCB after a quench in GGE

We consider that the sytem at t=0 is described by the equation ??, where:

$$V_i = V\cos(\pi i/2) \quad (11)$$

After the sudden turn off of the superlattice potential, an effective temperature can be approximeted by the relation:

$$\langle H_a \rangle_{GE}(T_{eff}) = \langle H_a \rangle_{GGE}, \qquad H_a : Hamiltonien \quad after \quad quench \quad (12)$$

The estemated temperature is given by:

$$k_B T_{est} B_{k(GGE)} \simeq J A_{k(GGE)} \quad (13)$$

where:

$$\langle O \rangle_{GGE} = \frac{1}{Z_{GGE}} Tr\left[O e^{-\sum_\alpha \lambda_\alpha I_\alpha}\right], \qquad Z_{GGE} = Tr\left[e^{-\sum_\alpha \lambda_\alpha I_\alpha}\right], \qquad \lambda_\alpha = ln\left(\frac{1 - \langle I_\alpha \rangle_0}{\langle I_\alpha \rangle_0}\right) \quad (14)$$

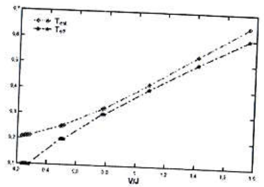

Fig. 3: *Estimated and effective temperature as a function of the superlattice potential in GGE for L=14.*

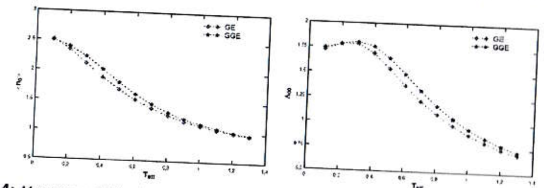

Fig. 4: *Momentum distribution and noise correlations peaks $\langle n_{k=0} \rangle$ and Δ_{00} as a function of T_{eff} both in GE and GGE for L=14.*

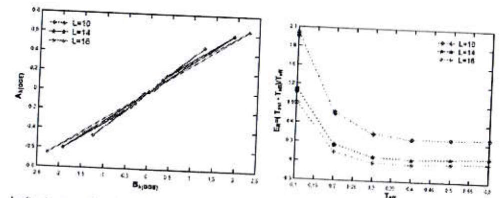

Fig. 5: *Left: A_k as a function of B_k in GGE for $V/J = 0.76$. Right: Relative error between the estimated temperature - equation ?? - and the approximated effective temperature - equation ?? - in GGE.*

4. Outlook

We test a fluctuation-dissipation relation (FDR) on the momentum distribution (which is strictly valid at equilibrium) in the case of the out-of-equilibrium GGE. It appears to be approximately satisfied by the GGE, allowing for the extraction of an effective temperature. This thermometry scheme offers an experimentally relevant estimator of the temperature in a quantum simulator, and it provides a temperature which, upon growing the size, appears to converge to the effective temperature estimated from the comparison with the GE. Further FDRs can be tested to probe the fluctuation properties in the GGE.

References

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6th World Congress on Physics, Paris from 13-14 May 2019, France.